Two-temperature theory for a heated semi-infinite solid by a pulsed laser radiation

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Abstract In this paper, the two-temperature thermoelasticity model is proposed to a specific problem of a thermoelastic semi-infinite solid. The bounding plane surface of the semi-infinite solid is considered to be un-der a non-Gaussian laser pulse. Generalized thermoelasticity analysis with dual-phase-lags is taken into account to solve the present problem. Laplace transform and its inversion techniques are applied and an analytical solu-tion as well as its numerical outputs of the field variables are obtained. The coupled theory and other generalized theory with one relaxation time may be derived as special cases. Comparison examples have been made to show the effect of dual-phase-lags, temperature discrepancy, laser-pulse and laser intensity parameters on all felids. An additional comparison is also made with the theory of thermoelasticity at a single temperature.

Keywords: Thermoelasticity; Two-temperature; Non-Gaussian laser pulse; Laser intensity; Phase-lags

1 Introduction

The dynamic theory of thermoelasticity has revolted much significance in current times. It has found enforcements in various engineering fields such as geothermal engineering, rising energy element accelerators, nuclear reac-tor designing, etc. The equation of heat conduction in the classical coupled thermoelasticity theory is parabolic in nature and thus predicts unlimited speed of the heat propagation of thermal waves. Obviously, this contrasts with physical observations. In the past four decades, attention is on theo-ries which assume a finite speed for the thermal signals.

The problems of heat and thermal sources behave in thermoelastic bod-ies are the ones of mathematical concern and physical importance. Using the uncoupled thermoelasticity theory, the problem of dynamic heat source is considered by Danilovskaya [1]. The problem of moving and instanta-neous heat sources in semi-infinite and infinite spaces were investigated by Eason and Sneedon [2], Nowacki [3], and others. Also, based on the ther-moelasticity coupled theory, Dhaliwal and Singh [4] provided a short time approximation subjected to a point heat source within an infinite space.

Biot who is the first investigator that introduced a coupled theory to overcome first shortcoming of classical uncoupled thermoelasticity theory [5]. This means that there is an additional shortcoming still in need to be modified. The second shortcoming is that the equation of heat is parabolic and depending on Fourier's law of heat conduction. So, Lord and Shul-man [6] and Green and Lindsay [7] presented their models to overcome this shortcoming. Additional modification to coupled theory is proposed by Tzou [8] and known as a dual-phase-lag (DPL) thermoelasticity model. Fourier's law in the DPL model is omitted and instead two different temper-atures for the heat flux and temperature gradient are introduced. Different thermoelasticity models have been presented and compared to discuss the three-dimensional thermal shock plate problem by Zenkour [9].

The two-temperature theory (2TT) was widely applied to predict tem-perature distributions in electrons and phonon in laser processing of very short metals. A 2TT of heat conduction in deformable bodies is presented by Chen and Gurtin [10] and Chen *et al*. [11,12]. This theory contains two distinct conductive and thermodynamic temperatures. The two tem-peratures are in general different for time-dependent mediums and this irrespective of inclusion of a heat supply. However, they are the same with as neglecting the heat supply. In particular, the difference between these temperatures is proportional to the heat supply for time-independent problems [13]. The wave propagation in 2TT has been presented by Warren and Chen [14]. Zenkour and his colleagues [15–20] have dealt with the 2TT to investigate many problems in thermoelasticity theory.

Throughout the last four decades, lasers have been extensively used for materials processing. As several applications depend on the thermal effects of laser-material interactions, it becomes very essential to get information about the temperature fields as a functions of materials properties and processing parameters. Understanding the effects and interactions of laser energy on matters is essential to developing new applications. The laser energy interactions in material are mostly characterized as thermal or photochemical. The thermal reaction of the laser comes about by absorbing the laser energy by the objective material.

Pulsed laser irradiation is utilized over a wide spectrum of materials processing employments. Wood *et al*. [21] dealt with the pulsed laser treat-ment of semiconductors. The stress wave created by laser pulses has been studied by Wang and Xu [22] in a semi-infinite medium. They take un-der consideration both non-Fourier effect in heat conduction and coupling effect of temperature and strain rate. It is to be noted that if a pulsed laser irradiates a metal surface the characteristic elastic waveforms will be generated. The significance of thermal diffusion to thermoelastic wave generation has been studied by McDonald [23]. Allam and Abouelregal discussed thermoelastic waves created by pulsed laser and varying heat of microbeam resonators [24].

The aim of the present paper is to investigate the created temperature and stress components in a thermoelastic half-space. Governing differential equations will be derived based upon the 2TT with phase-lags. The boundary surface is heated by a non-Gaussian laser beam. Analytical solution is outlined in the Laplace transform domain. Numerical solu-tion is then obtained by adopting inversion of the Laplace transform. So, the conductive and thermodynamic temperatures as well as other consti-tutive stress-strain distributions will be obtained numerically. Effects of two-temperature, laser-pulse, and laser intensity parameters are all inves-tigated.

2 Two-temperature with phase-lags model

The heat flux vector $\vec{q}$ according to the classical Fourier law of thermoelasticity may be contacted to temperature gradient by

$$\vec{q} = -K\nabla\theta, \tag{1}$$

in which K denotes thermal conductivity of a solid and $\theta = T - T_0$ is the thermodynamical temperature, T denotes the absolute temperature of medium, T_0 denotes reference temperature of body chosen such $|\theta/T_0| \ll 1$. The equation of heat conduction is presented as

$$\rho C_E \frac{\partial\theta}{\partial t} + \gamma T_0 \frac{\partial}{\partial t}(\nabla\cdot\vec{u}) = -\nabla\cdot\vec{q} + Q, \tag{2}$$

where ρ denotes the density, C_E denotes specific heat, $\vec{u}$ represents displacement vector, $\gamma = (3\lambda + 2\mu)\,\alpha_t$, λ and μ represent Lame's properties, α_t denotes coefficient of linear thermal expansion and Q denotes intensity of heat source.

The modification of the classical thermoelasticity theory is proposed by Tzou in which Fourier law is substituted by an approximation of heat flux vector as (using Einstein's summation convention) [25]

$$\vec{q}\,(x, t + \tau_q) = -K\nabla\theta\,(x, t + \tau_\theta), \tag{3}$$

in which τ_θ represents the phase lag of heat flux, and τ_q represents phase lag of temperature gradient. Equation (3) becomes [23]

$$\left(1 + \tau_q \frac{\partial}{\partial t}\right)\vec{q} = -K\nabla\left(1 + \tau_\theta \frac{\partial}{\partial t}\right)\theta. \tag{4}$$

In classification of real material into simple and non-simple materials Chen and Gurtin have presented a theory of non-simple materials for which ther-modynamics and conductive temperatures are not identical unlike simple materials for which they are identical [10]. This theory was further extended to deformable bodies by Chen *et al.* [11, 12]. Considering isotropy and the linearity, for such materials, they have shown that the two temperatures are related by the relation [26]

$$\phi - \theta = b\nabla^2\phi, \tag{5}$$

where ϕ denotes the conductive temperature, θ denotes thermodynamic temperature, and $b > 0$ is temperature discrepancy factor.

Now, in isotropic medium, it is assumed the following new generalized heat conduction equation:

$$\left(1+\tau_q \frac{\partial}{\partial t}\right)\vec{q} = -K\left(1+\tau_\theta \frac{\partial}{\partial t}\right)\nabla\phi . \tag{6}$$

Taking divergence of both sides of Eq. (6), one gets

$$\left(1+\tau_q \frac{\partial}{\partial t}\right)(\nabla\cdot\vec{q}) = -K\left(1+\tau_\theta \frac{\partial}{\partial t}\right)\nabla^2\phi . \tag{7}$$

The generalized heat conduction equation with two temperatures in case of non-simple medium using Eqs. (2) and (7) takes the form [26]

$$K\left(1+\tau_\theta \frac{\partial}{\partial t}\right)\nabla^2\phi = \left(1+\tau_q \frac{\partial}{\partial t}\right)\left(\rho C_E \frac{\partial\theta}{\partial t} + \gamma T_0 \frac{\partial}{\partial t}(\nabla\cdot\vec{u})\right) \tag{8}$$

This equation is the generalized heat conduction with the temperatures θ and ϕ. The classical one-temperature theory (1TT) will be given by setting $b \to 0$ and $\phi \to \theta$.

Equations of motion without body forces are given by

$$(\lambda+\mu)\nabla\cdot(\nabla\cdot u_i)+\mu\nabla^2 u_i-\gamma\theta = \rho\ddot{u}_i . \tag{9}$$

where double overdot on u denotes the acceleration vector. The constitutive relations take the forms

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - \gamma\theta)\,\delta_{ij} , \tag{10}$$

where σ_{ij} represent the stress components and δ_{ij} is Kronecker's delta.

In what follow some cases can be deduced from Eqs. (8)–(10) as:

- The coupled thermoelasticity theory with two-temperature (CTE): $\tau_q = \tau_\theta = 0$.
- The two-temperature thermoelasticity model with one relaxation time (LS): $\tau_q > 0$ and $\tau_\theta = 0$.
- The two-temperature thermoelasticity theory with phase-lags (DPL): $\tau_q > \tau_\theta \geq 0$. The one-temperature thermoelasticity theories (CTE, LS and DPL): $b \to 0$ and $\phi \to \theta$.

3 Formulation

It is considered a conductive thermoelastic isotropic solid that occupies a half-space $x \geq 0$. The half-space is uniformly irradiated by the bounding plane ($x = 0$) by a laser pulse with non-Gaussian profile. The system is initially quiescent with fields depending on x and t.

The displacement field for a 1D medium has $u_x = u(x, t)$ and $u_y = u_z = 0$ with the strain $e = e_{xx} = \partial u/\partial x$. The constitutive relation will be

$$\sigma_{xx} = \sigma = (\lambda+2\mu)\, e - \gamma\theta . \tag{11}$$

The dynamic equation is expressed as

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial \theta}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} . \tag{12}$$

or may be re-written as

$$\frac{\partial^2 \sigma}{\partial x^2} = \rho\frac{\partial^2 e}{\partial t^2} . \tag{13}$$

The heat conduction and thermodynamic heat formula is given by

$$\phi - \theta = b\frac{\partial^2 \phi}{\partial x^2} . \tag{14}$$

So, Eq. (8) is expressed as

$$K\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\frac{\partial^2 \phi}{\partial x^2} = \left(1+\tau_q\frac{\partial}{\partial t}\right)\left[\rho C_E\frac{\partial}{\partial t}\left(\phi - b\frac{\partial^2 \phi}{\partial x^2}\right) + \gamma T_0\frac{\partial e}{\partial t} - Q\right] . \tag{15}$$

The following dimensionless parameters:

$$\{x', u'\} = c_1\eta\{x, u\}, \quad \{t', \tau_\theta', \tau_q'\} = c_1^2\eta\{t, \tau_\theta, \tau_q\}, \quad \{\theta', \phi'\} = \frac{\gamma}{\rho c_1^2}\{\theta, \phi\}, \quad \sigma' = \frac{\sigma}{\rho c_1^2}, \quad Q' = \frac{Q}{K c_1^2 \eta^2 T_0} \tag{16}$$

may be introduced here in which $c_1 = \sqrt{(\lambda+2\mu)/\rho}$ denotes the longitudinal wave speed and $\eta = \rho C_E/K$ denotes the thermal viscosity. So, Eqs. (11) and (13)-(15) can be transformed into the dimensionless forms:

$$\sigma = e - \theta , \tag{17}$$

$$\frac{\partial^2 e}{\partial x^2} - \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 e}{\partial t^2} , \tag{18}$$

$$\phi-\theta=\beta\frac{\partial^2\phi}{\partial x^2}\,, \tag{19}$$

$$\left(1+\tau_{\theta}\frac{\partial}{\partial t}\right)\frac{\partial^2\phi}{\partial x^2}=\left(1+\tau_{q}\frac{\partial}{\partial t}\right)\left[\frac{\partial}{\partial t}\left(\phi-\beta\frac{\partial^2\phi}{\partial x^2}\right)+\varepsilon\frac{\partial e}{\partial t}-Q\right], \tag{20}$$

where $\varepsilon = \gamma^2T_0/\rho^2C_E c_1^2$ and $\beta = bc_1^2\eta^2$. Now, let us consider a medium with uniformly heated by a laser pulse with non-Gaussian profile [24] as

$$I(t)=\frac{L_0 t}{t_p^2}e^{-t/t_p}\,, \tag{21}$$

in which t_p represents a characteristic time of laser-pulse and L_0 denotes laser intensity. The conduction heat transfer in the medium is represented as a 1D problem with an energy source, $Q(x,t)$ near the surface, i.e.,

$$Q(x,t)=\frac{R_a}{\delta_1}e^{(x-h/2)/\delta_1}I(t)=\frac{R_aL_0}{\delta_1 t_p^2}te^{(x-h/2)/\delta_1-t/t_p}\,, \tag{22}$$

where δ_1 represents absorption depth of heating energy and R_a represents surface reflectivity [27]. It is clear that the laser pulse is lying on the medium surface (x = 0). So, the energy source will be

$$Q(t)=\frac{R_aL_0}{\delta_1 t_p^2}te^{-h/(2\delta_1)-t/t_p}\,. \tag{23}$$

4 Problem conditions

The problem will be discussed under the following proper initial and bound-ary conditions:

$$\theta(x,t)=\phi(x,t)=u(x,t)=\frac{\partial\theta(x,t)}{\partial t}=\frac{\partial\phi(x,t)}{\partial t}=\frac{\partial u(x,t)}{\partial t} \quad \text{at } t=0\,. \tag{24}$$

Other thermomechanical boundary conditions on x=0 of the half-space will be expresses as

- Thermal boundary condition:
  The boundary plane x = 0 is subjected to a thermal shock. That is

$$\theta(0,t)=\theta_0H(t)\,, \tag{25}$$

  where $H(t)$ is called the Heaviside's unit step function and θ_0 is constant.

- Mechanical boundary condition:
  The boundary plane $x = 0$ is considered to be traction free,

$$\sigma(0, t) = 0 . \tag{26}$$

5 Solution in Laplace space

Laplace transform with variable t may be applied for Eqs. (17)–(20) to get a transformed system as

$$\bar{\sigma} = \bar{e} - \bar{\theta} , \tag{27}$$

$$\left(\frac{d^2}{dx^2} - s^2\right)\bar{e} = \frac{d^2\bar{\theta}}{dx^2} , \tag{28}$$

$$\bar{\theta} = \bar{\phi} - \beta \frac{d^2\bar{\phi}}{dx^2} , \tag{29}$$

$$\frac{d^2\bar{\phi}}{dx^2} = \alpha_1 \left(\bar{\theta} + \varepsilon \bar{e}\right) - \bar{G}(s) , \tag{30}$$

where

$$\bar{G}(s) = \frac{R_a L_0 e^{-h/(2\delta_1)}}{\delta_1 t_p^2 K c \left(1+s\tau_\theta\right)}\left(\frac{t_p \tau_q}{1+s t_p} + \frac{t_p\left(t_p-\tau_q\right)}{\left(1+s t_p\right)^2}\right) , \quad \alpha_1 = \frac{s\left(1+s\tau_q\right)}{1+s\tau_\theta} . \tag{31}$$

Eliminating $\bar{\theta}$ and $\bar{e}$ from equations (27)–(30), one obtains

$$\left(\frac{d^4}{dx^4} - A\frac{d^2}{dx^2} + B\right)\bar{\phi} = 0 , \tag{32}$$

$$A = \frac{s^2\left(1+\alpha_1\beta\right) + \alpha_1\left(1+\varepsilon\right)}{1+\alpha_1\beta\left(1+\varepsilon\right)} , \quad B = \frac{s^2\alpha_1}{1+\alpha_1\beta\left(1+\varepsilon\right)} . \tag{33}$$

The solution of Eq. (32) is expressed as

$$\bar{\phi} = A_1 e^{-m_1 x} + A_2 e^{-m_2 x} , \tag{34}$$

where A_1 and A_2 are parameters of s. The solution of $\bar{\phi}$ is used in Eq. (29) to get the solution of $\bar{\theta}$

$$\bar{\theta} = \left(1-\beta m_1^2\right) A_1 e^{-m_1 x} + \left(1-\beta m_2^2\right) A_2 e^{-m_2 x} . \tag{35}$$

The above two solutions are used in Eq. (30) to get the solution of e as

$$\bar{e} = \bar{F}(s) + \Omega_1 A_1 e^{-m_1 x} + \Omega_2 A_2 e^{-m_2 x}, \tag{36}$$

where

$$\bar{F}(s) = \frac{\bar{G}(s)}{\varepsilon\alpha_1}, \quad \Omega_i = \frac{m_i^2(1+\alpha_1\beta)-\alpha_1}{\varepsilon\alpha_1}, i = 1, 2. \tag{37}$$

Substituting Eqs. (35) and (36) into Eq. (27), one gets

$$\bar{\sigma} = \bar{F}(s) + \left(\Omega_1+\beta m_1^2-1\right) A_1 e^{-m_1 x} + \left(\Omega_2+\beta m_2^2-1\right) A_2 e^{-m_2 x}. \tag{38}$$

The substitution of Eq. (13) and using dimensionless variables as well as Laplace transforms gives the displacement as

$$\bar{u} = \frac{1}{s^2}\frac{d\bar{\sigma}}{dx} = -\frac{m_1}{s^2}\left(\Omega_1+\beta m_1^2-1\right) A_1 e^{-m_1 x} - \frac{m_2}{s^2}\left(\Omega_2+\beta m_2^2-1\right) A_2 e^{-m_2 x}. \tag{39}$$

Also, the thermomechanical conditions in Laplace space $\bar{\theta}(0, s) = \theta_0/s$ and $\bar{\sigma}(0, s) = 0$ with the help of Eqs. (35) and (38), gives

$$A_1 = \frac{\bar{F}_s\left(\beta m_2^2-1\right) - \theta_0\left(\Omega_2+\beta m_2^2-1\right)}{s\left[\Omega_1\left(1-\beta m_2^2\right) - \Omega_2\left(1-\beta m_1^2\right)\right]}, \quad A_2 = \frac{\bar{F}_s\left(\beta m_1^2-1\right) - \theta_0\left(\Omega_1+\beta m_1^2-1\right)}{s\left[\Omega_2\left(1-\beta m_1^2\right) - \Omega_1\left(1-\beta m_2^2\right)\right]}. \tag{40}$$

Here, a numerical inversion technique for the Laplace transforms, depending on the Fourier series expansion introduced by Durbin [28], is established. In this technique, all field quantities can be determined by using a numerical inversion method based on a Fourier series expansion [29]. The inverse $f(t)$ of Laplace transform $\bar{f}(s)$ is approximated by the formula

$$f(t) = \frac{e^{\zeta t}}{t_1}\left[\frac{1}{2}\bar{f}(\zeta) + \mathrm{Re}\sum_{k=0}^{N}\bar{f}\left(\zeta+\frac{ik\pi}{t_1}\right)e^{ik\pi t/t_1}\right], \quad 0 \le t \le t_1, \tag{41}$$

where N must be chosen such that

$$e^{\zeta t}\,\mathrm{Re}\left[e^{iN\pi t/t_1}\bar{f}\left(\zeta+\frac{iN\pi}{t_1}\right)\right] \le \varepsilon_1, \tag{42}$$

in which ε_1 and ζ represent small positive numbers [29].

6 Numerical results and discussions

To discuss effects of 2TT parameter, laser-pulse, and laser intensity coef-ficients on wave propagation we used copper material with the following properties at $T_0 = 293$ K:

K = 368 N/Ks, $\alpha_t = 1.78 \times 10^{-5}$ 1/K, $C_E = 383.1$ m^2/K,

ρ = 8954 kg/m^3, $\lambda = 7.76 \times 10^{10}$ N/m^2, $\mu = 3.86 \times 10^{10}$ N/m^2. Results are computed for $x(0 \le x \le 1)$ at small interval of time

$t_0 = 0.15$. All variables will be displayed in Figs. 1–20. It is assumed that $\delta_1 = 0.01$, $\tau_0 = 0.02$, $R_a = 0.5$, and $h = 0.1$. The distributions of conductive and dynamical temperatures, stress, strain, and displacement may be obtained in terms of different parameters such as t, x, 2TT parameter *beta*, time of laser-pulse t_p, and laser intensity L_0. Here, laser intensity is assumed to be of the form $L_0 = \xi \times 10^{11}$ J/m^2, where ξ is laser intensity parameter. Numerical computations are carried out for the following four cases:

Firstly, Figs. 1–5 show the distributions of displacement (u), thermodynamical temperature (θ), conductive temperature (ϕ), stress (σ), and strain

e for different 2TT parameter β to highlight the effect of β on all variables. The value $\beta = 0$ points the old situation (1TT) while $\beta = 0.02$ or 0.04 indicates the 2TT. Here, one puts $\tau_q = 0.2$, $\tau_\theta = 0.1$, $\xi = 0.1$, and $t_p = 2$. The wave-amplitude of the displacement u decreases as β increases. For $x > 0.1$, the thermodynamical temperature θ decreases with the increase in β. Also, β is increasing as the conductive temperature ϕ is decreasing in $0 < x < 0.3$ and is increasing in interval $0.3 < x < 1$. In most positions, strain e increases as β increases while stress σ increases in $0 < x < 0.28$ and decreases in $0.28 < x < 1$. This shows the difference between the 1TT of DPL model ($\beta = 0$) and the 2TT ($\beta = 0.02$ or 0.04). The figures attend that this parameter has significant effects on all variables.

In the second case, Figs. 6–10 plot the field quantities with a characteristic time of laser-pulse t_p to stand on its effect on all variables. Here, the temperature discrepancy parameter β remains constant ($\beta = 0.02$) and the lags are setting as $\tau_q = 0.2$, and $\tau_\theta = 0.1$. The figures illustrate that t_p causes a difference between results in context of 2TT. The conductive heat, strain and the thermodynamic heat decrease with the increase in t_p while both displacement and stress are increasing. This means that t_p has a significant effect on all variables.

In the third case, Figs. 11–15 plot field quantities for different values of the laser intensity parameter ξ to highlight the effect of ξ on all variables.

It is seen that ξ has significant effects with the constancy of β = 0.02 and t_p = 0.2. It is clear that the nature of variations of all variables for laser intensity parameter is significantly different. All fields are increasing when ξ is increasing.

Finally, various values of the dual-phase-lags (DPLs) of heat flux and temperature gradient τ_q and τ_θ, respectively, are considered. The graphs in Figs. 16–20 denote the curves predicted by various theories. The coupled theory (CTE) ($\tau_q = \tau_\theta = 0$), Lord-Shulman (LS) theory ($\tau_q = 0.2$, $\tau_\theta = 0$), and generalized theory of thermoelasticity proposed by Tzou (DPL) ($\tau_q = 0.2$, $\tau_\theta = 0.1$) are all considered as special cases. It can be noted that the PL parameters have great effects on distributions of all variables. The field variables propagate as waves with finite velocity in the medium. The CTE model is different in comparison with those of other theories. It is apparent in these figures the fact that in theories of generalized thermoe-lasticity (DPL and LS) spreads waves at finite speeds. The response to the three theories is generally quite similar.

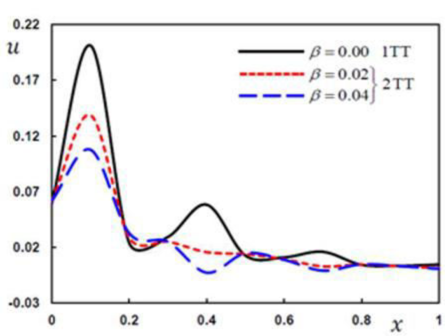

Figure 1: Variation of displacement u ***versus*** two-temperature parameter β.

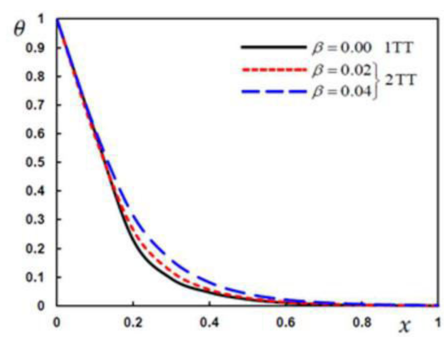

Figure 2: Variation of thermodynam-ical temperature θ ***versus*** two-temperature parameter β.

In general, the amplitude of the wave of the displacement u is decreasing along the distance x. The thermodynamical (θ) and conductive (ϕ) temper-atures are directly decreasing along the distance (x). The thermodynamical temperature (θ) may be vanish at $x = 1$ while conductive temperature (ϕ) may be tending to the value 0.1. The stress (σ) is no longer decreasing and has its absolute minimum at $x = 1$. However, the strain (e) is no longer increasing and has its absolute maximum at $x = 1$. The wave-amplitude of displacement (u) is decreasing along the distance and may be vanished at $x = 1$.

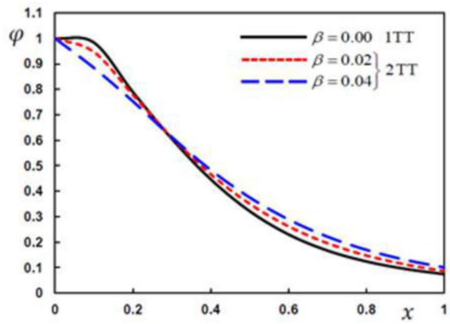

Figure 3: Variation of conductive temperature ϕ ***versus*** two-temperature parameter β.

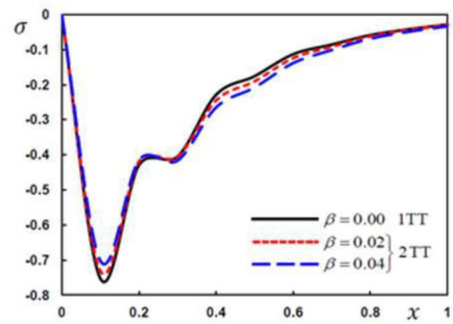

Figure 4: Variation of thermal stress σ ***versus*** two-temperature parameter β.

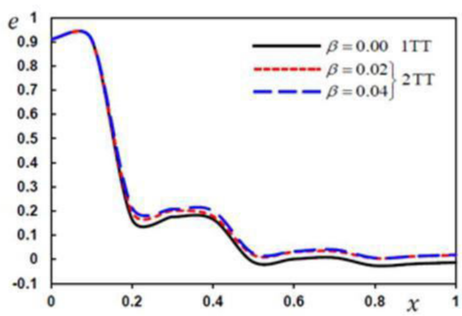

Figure 5: Variation of strain e ***versus*** two-temperature parameter β.

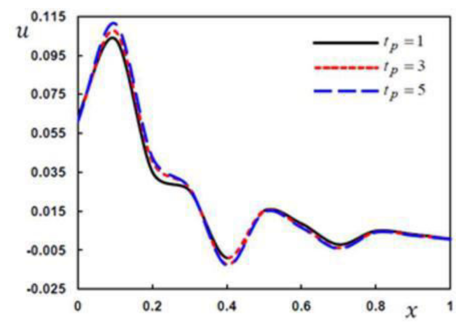

Figure 6: Variation of displacement u ***versus*** time of laser-pulse t_p .

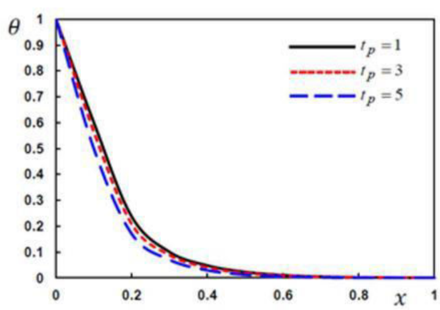

Figure 7: Variation of thermodynamical temperature θ ***versus*** time of laser-pulse t_p .

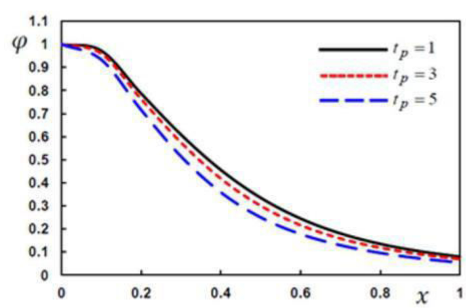

Figure 8: Variation of conductive temperature ϕ ***versus*** time of laser-pulse t_p .

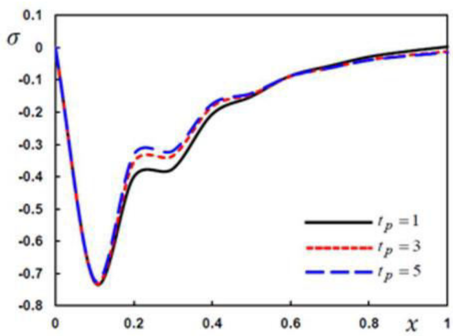

Figure 9: Variation of thermal stress σ versus time of laser-pulse t_p.

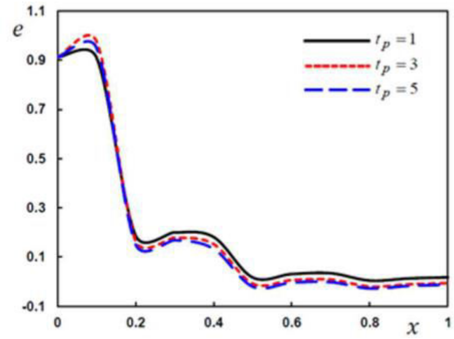

Figure 10: Variation of strain e versus time of laser-pulse t_p .

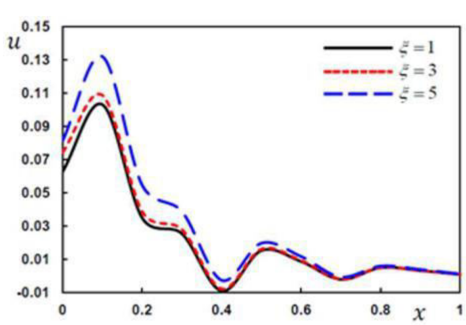

Figure 11: Variation of displacement u ***versus*** laser intensity parameter ξ.

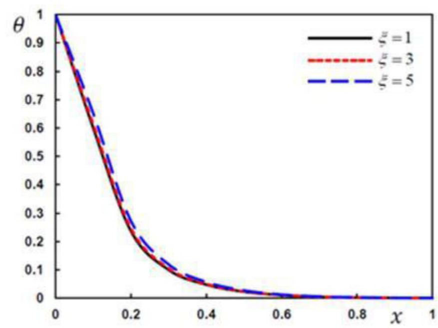

Figure 12: Variation of thermodynamical temperature θ ***versus*** laser intensity parameter ξ.

7 Concluding remarks

In this article, the two-temperature thermoelasticity analysis is constructed. The two-temperature in context of dual-phase-lags model is adopted to solve this problem. The effects of time of laser-pulse, laser intensity and phase lags τ_θ and τ_q as well as the two-temperature parameter on all vari-ables are investigated.

The results obtained from the above analysis can be briefed as:

1. The influence of phase-lags parameters plays a significant role in all variables.
2. All field variables are significantly depending on the two-temperature parameter.

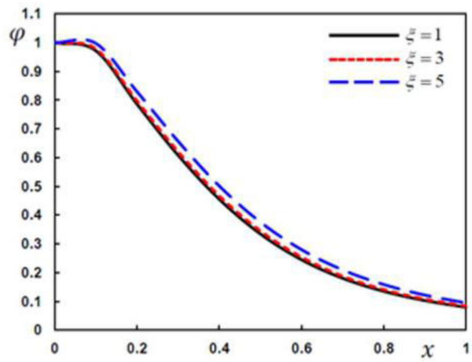

Figure 13: Variation of conductive temperature ϕ ***versus*** laser intensity parameter ξ.

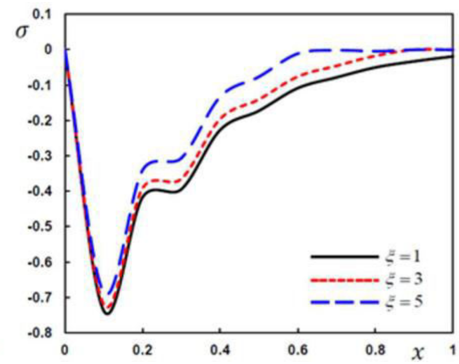

Figure 14: Variation of thermal stress σ versus laser intensity parameter ξ.

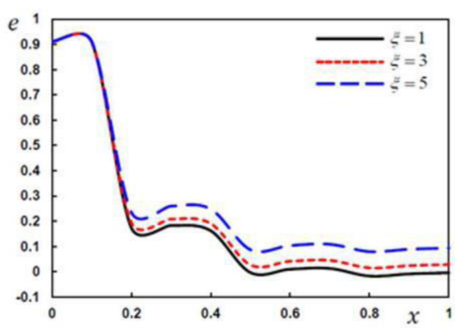

Figure 15: Variation of strain e versus laser intensity parameter ξ.

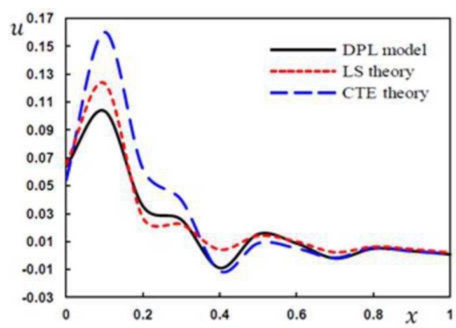

Figure 16: Variation of displacement u according to different theories of thermoelasticity.

3. According to the present two-temperature theory, a new classification for materials according to their fractional parameter is constructed.
4. Theories of coupled thermoelasticity and generalized thermoelasticity with one relaxation time can be obtained as limited cases.
5. From our results, the two-temperature theory is considered as an improvement in deducing elastic materials.
6. The properties of a body depend largely on time duration of a laser pulse and laser intensity of applied source. Therefore, the presence of non-Gaussian laser pulse in the current model is of significance.

Finally, the laser contains a variety of applications in modern day tech-

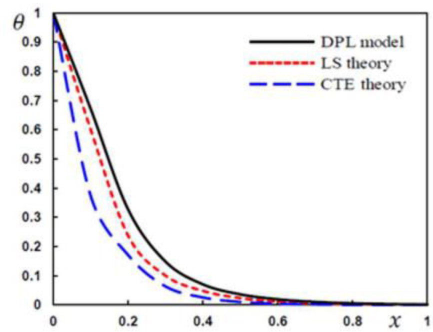

Figure 17: Variation of thermodynamical temperature θ according to different theories of thermoelastic-ity.

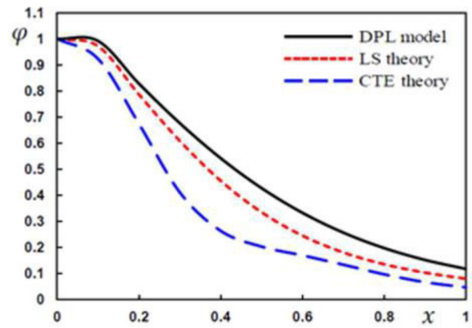

Figure 18: Variation of conductive temperature ϕ according to different theories of thermoelasticity.

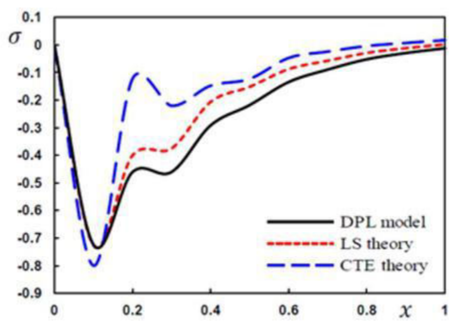

Figure 19: Variation of thermal stress σ according to different theories of thermoelasticity.

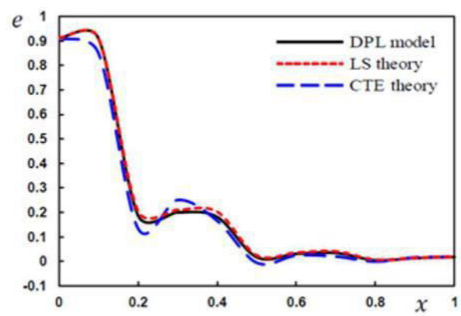

Figure 20: Variation of strain e according to different theories of thermoe-lasticity.

nology, mainly due to its ability to produce high-power beams. Using this concentrated energy, any known substance can be heated, defrosted or evaporated. Now, laser applications include drilling, welding, cutting, heat treatment of metals, machining of brittle or refractory materials, and fabrication of electronic components, medical surgery, and the production of charged particles. Most of the theoretical work on laser heat transfer to date has focused on solving the classical equation of heat conduction for a moving or stationary semi-infinite mediums.

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